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INTRODUCTION

All students need a high-quality mathematics program designed to prepare them to choose from among a full range of career paths. California's Mathematics Task Force (1995) called for a rigorous and challenging mathematics program for every student—a complete program that reflects a balance of computational and procedural basic skills, conceptual understanding, and problem solving. Yet the current mathematics achievement of many students is unacceptably low (Reese et al. 1997). Educators are challenged to expect more from students in mathematics, to raise the bar for student achievement through more rigorous curriculum and instruction, and to provide the support necessary for all students to meet or exceed California's mathematics standards.

To compete successfully in the worldwide economy and to participate fully as informed citizens, today's students must have a high degree of comprehension of mathematics. The goal of the State Board of Education is that all students will attain California's mathematics standards for kindergarten through grade twelve (hereinafter referred to as "the standards") and that many will be inspired to pursue advanced studies in mathematics.

This framework is predicated on the belief that proficiency in mathematics is a consequence of sustained student effort and effective instruction. All students are capable of understanding mathematics, given the opportunities and encouragement to do so.

What's New in the 2005 Framework?

The most important feature of this framework is its focus on the content of the mathematics standards adopted by the California State Board of Education. The goal of accelerating student progress through a standards-based program has a significant impact on the entire curriculum. As specified by the standards, much of the content of the mathematics curriculum has been shifted into earlier grades, and mathematics instruction in kindergarten through grade seven is substantially strengthened. These changes necessitate a more substantive, rigorous, and demanding curriculum and more systematic instruction that will better serve California students.

A goal of this framework is to prepare all children to study algebra by eighth grade. In 2000, legislation was enacted requiring students to complete Algebra I, as a condition of receiving a high school diploma (*Education Code* Section 51224.5(b)). To support this legislative goal for all students, the framework includes guidance on the design of two new types of specialized instructional mathematics materials for students having difficulty achieving at grade level: a Mathematics Intervention Program for strategic and intensive students in grades four through seven and Algebra Readiness materials for students in grade eight or above who are not ready to take a course in algebra. If, when and how to used these materials, to meet the needs of students, should be a collective decision involving teachers, counselors, administrators, parents, and students. Intervention materials are intended to accelerate student achievement, so that students can be successful in the basic grade level program and complete Algebra I. As previously stated, Algebra I in

eighth grade is the goal, and the availability of Algebra Readiness materials should not be used to restrict students, who are ready for Algebra I in eighth grade, from enrolling in an Algebra I course.

An important theme stressed throughout this framework is the need for a balance in emphasis on computational and procedural skills, conceptual understanding, and problem solving. This balance is defined by the standards and is illustrated by problems that focus on these components individually and in combination. All three components are essential.

A General Overview: Purpose, Organization, and Audience

The purpose of this framework is to guide the curriculum development and instruction that teachers provide in their efforts to ensure that all students meet or exceed the mathematics standards. The framework provides a context for implementing the standards in the form of guidelines for the design of curricula, instructional materials, instructional practices, and staff development. Building on the standards, the framework addresses the manner in which all students in California public schools can best meet the standards. *All students* includes those performing at, below, and above grade level; English learners; special education students; and others with special learning needs.

More specifically, the framework*:*

- Describes *guiding principles and key components* of an effective mathematics program (Chapter 1)
- Presents the essential skills and knowledge expected of students in mathematics as described in the *Mathematics Content Standards for California Public Schools*

and illustrated by sample problems (Chapter 2)

- Describes *special considerations and emphases for each grade level to ensure student success* consistent with the mathematics standards and statewide testing program (Chapter 3)
- Provides guidance, based on current research, regarding *instructional strategies* and sample lessons that can be effective in ensuring that every child will meet or exceed grade-level standards in mathematics (Chapter 4)
- Guides the development of appropriate *assessment* methods (Chapter 5)
- Suggests specific strategies to ensure *access* to appropriately challenging curriculum for special needs students (Chapter 6)
- Describes the *responsibilities* that all stakeholders must uphold for effective implementation of a rigorous and coherent kindergarten through grade twelve mathematics curriculum (Chapter 7)
- Suggests guidelines for both pre-service teacher preparation and in-service *professional development* (Chapter 8)
- Provides guidance on the use of *technology* in mathematics instruction (Chapter 9)
- Specifies requirements for *instructional resources,* including print and electronic learning resources (Chapter 10)

In short, the framework should be viewed as a critical tool for designing and implementing an effective mathematics program in kindergarten through grade twelve and for evaluating instructional resources.

The framework addresses two primary audiences: (1) educators; and (2)

developers of instructional resources. Educators include those involved in the day-to-day implementation of school mathematics programs—classroom teachers, school administrators, district personnel, school board members, and others responsible for curriculum and instruction. It also addresses other important audiences, such as parents and community members, including business and civic leaders, who have a vital stake in the success of California students in mathematics.

The following themes permeate the *Mathematics Framework:*

The framework:

- Builds on the mathematics standards and aligns them with curriculum, instruction, resources for instruction, and assessment, resulting in a coherent and pragmatic plan for achieving high levels of mathematics proficiency for all students. It also provides guidance for understanding the standards by identifying priorities within the standards and offering concrete examples of mathematical problems that demonstrate the concepts within the standards.

The framework:

- Emphasizes the importance of a balanced mathematics curriculum. In particular, the framework stresses the critical interrelationships among computational and procedural proficiency, problem-solving ability, and conceptual understanding of all aspects of mathematics, from the simplest calculations to the most sophisticated problem solving.

The framework:

- Addresses the needs of all learners, with no learner left out and no learner taught at the expense of another; emphasizes prevention over remediation, while

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recognizing the appropriateness of remediation when it is required; and provides suggestions for instructional strategies that may be used with students who are English learners, advanced learners, special education pupils, or at risk of failing mathematics.

The framework:

- Highlights the importance of mathematical reasoning. The mathematical reasoning standards are different from the others in that they do not represent a specific content area. Mathematical reasoning cuts across all strands. It characterizes the thinking skills that students can carry from mathematics into other disciplines. Constructing valid arguments and criticizing invalid ones are inherent in doing mathematics.

The framework:

- Stresses the importance of frequently assessing student progress toward achieving the standards. Students cannot afford to wait for a year-end test; rather, they should be assessed frequently throughout the year to determine their progress toward achieving the standards. Teachers, students, and parents and guardians need some daily or weekly indication of the extent to which the standards are likely to be met.

The framework:

- Avoids oversimplified guidance on either content or pedagogy in favor of guidelines on effective instruction derived from reliable research.